Problem Solving Oriented Mathematics Curriculum

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ABSTRACT

Most of students think mathematics as memorising not understanding. They usually use formulas for solving mathematics questions so that when facing non-routine questions they feel frustrating how to solve it. As a mathematics teacher feel dilemma because new curriculum, K13 (curriculum 13) requires them to teach more topic compared than previous curriculum. Consequently, they teach mathematics using textbooks given by government. Due to the limited time to teach many topics, teachers only focus on finishing the topics rather than stimulate students' problem-solving skills. Therefore, there may be a specific curriculum that can be developed regarding students' problem solving skills in learning mathematics. Using some literature review, a framework of problem solving oriented mathematics curriculum (PSOMC) was developed. PSMOC has seven principles: Designing, generating, researching, hipotesising, deciding, communicating, and reflecting.

Keywords: problem-solving skills, mathematics curriculum

INTRODUCTION

There was a situation when a teacher feels failed as a mathematics teacher. It based on authors observation when they worked as private teachers. One of students was a high-achieving student who was always positioned as the best three in her classroom and is fond of mathematics. She used to learn mathematics by memorising the formulas and was very diligent to solve any mathematical problems from her textbook. Due to her good marks in mathematics, one day, she was confident to take part in a mathematics competition held by a university in the city. She passed the first round, which was multiple choice questions but failed on the second round, which tested more challenging essay questions. While joining another competition in mathematics, the result remained the same: she was not able to tackle the difficulty of the essay questions which require problem-solving ability. She kept protesting to us by saying that, "Those problems were too difficult. I never solved that kind of problems before. I did not know how to use my formulas to deal with those problems". Then, failing twice made her refuse to join any mathematical competitions anymore. Reflecting on this situation, we observed that being too focused on mathematical formulas might hinder students creative thinking to see the challenges

from different point of views. Besides, most of the tests which students solved or which were given by the teacher in the classroom consisted of routine problems which do not give a significant influence to develop their problem-solving skill.

Indonesia has developed a new curriculum called K13 (Curriculum 2013) which uses saintific approache as the main thing teachers have to do during learning and teaching process in the classroom. However, some teachers are likely to be confused how to apply it in their teaching especially a mathematics teacher. National curriculum mathematics requires teachers to teach many topics that spend much time because there are two mathematics, compulsary mathematics and specialisation mathematics (Kemendikbud, 2013). As a result, they only think how to finish all topics according to available time. They also focus on textbooks that contain mostly routine problems so that students are not accustomed to solving mathematics problems categorised non-routine and Higher Order Thinking Skills (HOTS) problems. Therefore, I assume that there may be a curriculum that can facilitate the students to improve their problem-solving ability effectively.

CURRICULUM

There are many definitions of curriculum that have been formulated by the experts. One of those is proposed by Glatthorn (1987, as cited in Stefan, 2010) as following:

> The curriculum is the plans made for guiding learning in schools, usually represented in retrievable documents of several levels of generality, and the actualisation of the plan in the classroom as experienced by the learners and as recorded by an observer, those experiences take place in a learning environment which also influences what is learned (p.13).

Authors prefer to adopt this definition as it depicts apparently the curriculum that we have experienced while we were a learner. Firstly, the plan for guiding learning represented in a number of documents explains the preparation materials that should be prepared by the teachers before starting the teaching and learning process, such as: lesson plan containing the objective of the lesson, teaching method and strategy, the assessment or feedback from the learners; and the syllabus containing the depth of the content that should be taught. These documents are made based on the policy created by the government, and the outcomes of the learning process would be

saved as the evaluation. Secondly, the actualisation of the plan in the classroom as experienced by the learners means that through the actual teaching and learning process, not all the intended plans in the curriculum could be applied well due to several reasons, such as the teaching strategy chosen might not support the students to grasp the concepts or the unavailability of technology as prepared. The interaction with the students in the classroom then will lead to some changes of the plans that have been arranged in order to achieve the goal of the curriculum. Thirdly, the learning environment influences what is learned means that the students would not just learn what has been planned in the curriculum but also will unconsciously learn some knowledge from their environment.

PROBLEM SOLVING

In the beginning, our ignorance about problem-solving led us to define it as a process while we solve any mathematical problems. However, Schoenfeld (1992) offered an intelligible definition that "problem solving is the process wherein students encounter the problem - a question for which they have no immediately apparent resolution, nor an algorithm that they can directly apply to get an answer" (Tripathi, 2009: p.168). A similar definition is also mentioned by Carl (1989) that

problem solving is defined as "a process where previously acquired data are used in a new and unknown situation" (p.389). Both definitions suggest that while facing a certain mathematical problem, the students cannot solve it directly by using any mathematical formulas or procedures rather, they have to read the problem carefully and try to engage with and make a sense of it until they come up with a way of solution by relating the existing knowledge that they have with the information embedded inside the problem.

Problem-solving is crucial in the mathematics curriculum. Anderson (2009) argued that the existence of problem-solving in many countries is to play one of the two roles: as a comprehensive goal or as a fundamental component of the mathematics curriculum. This statement leads to a question, what does actually problem-solving do which makes it so essential in the curriculum? Novotna et al. (2014) answered this question by saying that "problem solving is an indicator of the state of grasping concepts and ideas pupils are learning" (p.1). On the other hand, NCTM (2010) includes problem solving in one of the five process standards to learn mathematics. Others four standards are reasoning, communication, connection and representation. In addition, NCTM explains the usefulness of these five standards as follows:

- Solving problem is not only a goal of learning mathematics but also a major means of doing so
- Mathematical reasoning and proof offer powerful ways of developing and expressing insights about a wide range of phenomena
- Mathematical communication is a way of sharing ideas and clarifying understanding.
- When students connect mathematical ideas, their understanding is deeper and more lasting
- The ways in which mathematical ideas are represented is fundamental to how people understand and use those ideas (p.4).

Based on the explanations from NCTM, it could be said that problem solving is the centre of the processes to learn mathematics because other four standards are integrated while doing problem solving. While given a mathematical problem, the students firstly might represent the information inside the problem in some different ways through connecting that information with their existing knowledge. Next, they could do reasoning to choose the best strategy that leads them to find the solution. Afterwards, the solution could be shared with their peers and here they might reason again to justify their

methods of solution. Hence, it could be also claimed that problem solving copes the existence of other four standards. Then, if we improve students' problem-solving skill, could other four skills (reasoning skill, communicating skill, connecting skill and representing) develop as well?

Additionally, Lubiensky (2000) suggested that "instead of students complete meaningless exercises and memorize what the teacher tells them, why not have students learn key mathematical ideas while solving interesting problems" (p.456). On the one hand, I do agree with Lubiensky's statement that through solving a good problem, the students could improve their creativity and critical thinking ability (Kopka, 2010 as cited in Novotna, 2014) while exploring any useful information in the problem as their effort to get the solution, which will direct them to get a new knowledge. However, on the other hand, Lubieknsky's view of a problem as a useless exercise is too extreme, as I believe that the teachers have considered well every problem that they give to the students. No matter how easy a problem is, there must be positive effect for the students that solve it, for instance to recall her arbitrary things (see Hewitt, 1999) or related mathematical formulas.

In order to be successful in problem solving, the students should possess some

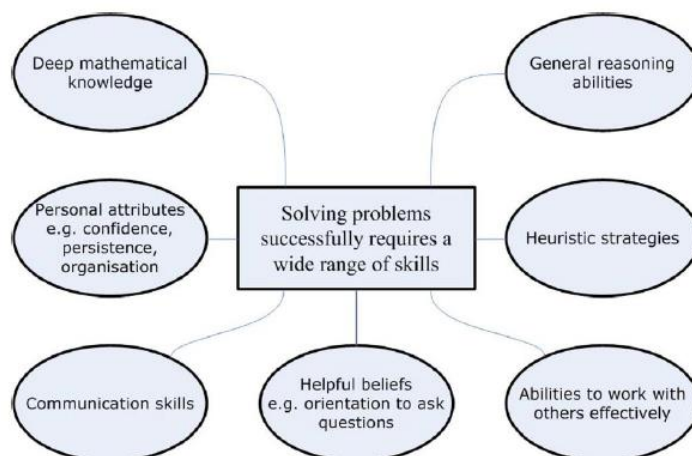factors. Stacey (2005) mentioned seven factors as presented in Figure 1 below.

Figure 1. Factors Contributing to Successful Problem Solving (Stacey, 2005)

The first factor mentioned by Stacey, having a deep mathematical knowledge, could be assumed as the prior knowledge that is already possessed by the students. This implies that the teacher should make sure that the students have sufficient prior knowledge to be able to understand all information inside the problems that they will solve. As mentioned by Anderson (1990, as cited in Dolmans et al., 1997) that "prior knowledge strongly influences the nature and amount of new information that can be processed" (p.186). Regarding heuristic strategy, Schoenfeld (1980) defined it as "a general suggestion or strategy, independent of any particular topic or subject matter that helps problem solvers approach and understand a problem, and effectively use their resources to solve it" (Williamson et.al, 2008: p.228). Many strategies in problem solving have been mentioned by the researchers, such as: looking for the patterns which is one of the fourteen strategies proposed by Polya (1957). Additionally, in her research, Novotna et al. (2014) characterised seven problem-solving strategies, namely: strategy of analogy; guess – check – revise; systematic experimentation; problem reformulation; solution drawing; working backwards; and use of graphs of functions. Polya (1957) also stated that the more problems solved by the students the more able they are to choose the appropriate heuristic strategies.

Additionally, the Ministry of Education, Singapore (MOES, 2006), claimed that, "the development of mathematical problem solving ability is dependent on five inter-related

components, namely, concepts, skills, processes, attitudes and metacognition" (p.6). It can be seen that both notions proposed by Stacey and MOES are in line. For example, the general reasoning abilities, communication skills, heuristic strategy and ability to work with others which were mentioned by Stacey, outline the process component proposed by MOES. Regarding the skills component, furthermore, MOES suggested seven skills to be mastered by the students, such as: procedural skills for numerical calculation, algebraic manipulation, spatial visualisation, data analysis, measurement, use of mathematical tools and estimation. The fourth factor seems appealing for me because not all of the students would have good spatial visualisation ability. As said by Lord (1985) and Piburn et al. (2002), "some of the students have natural visual ability that is different for each individual" (as cited in Safarin et al., 2013: p.1770). Then, would the students with good spatial visualisation ability be more successful in problem solving?

In the framework of its curriculum, MOES (2006) categorises attitude toward learning mathematics into five categories, namely: (1) beliefs about mathematics and its usefulness; (2) interest and enjoyment in learning mathematics; (3) appreciation of the beauty and power of mathematics; (4) confidence in

using mathematics; (5) perseverance in solving a problem. MOES (ibid.) also suggested that the learning experience plays a role in shaping students' attitude toward mathematics. On the other hand, Williamson et al. (2008) argued that students' belief could influence their problem-solving performances in which they decide any ideas of soltution. Furthermore, Middleton and Spanais (1999) also stated the importance of motivation in problem solving, as follow:

> When individuals engage in tasks in which they are motivated intrinsically they tend to exhibit a number of pedagogically desirable behaviours including increased time on task, persistence in the face of failure, more elaborate processing, the monitoring of comprehension, and selection of more difficult tasks, greater creativity and risk taking, selection of deeper and more efficient performance and learning strategies and choice of activity in the absence of extrinsic reward (p. 66).

Hence, it could be concluded that a positive attitude toward mathematics could be gained by the students as long as the teachers create a meaningful, fun and relevant learning situation.

Lester (2013) mentioned an issue with the relation between problem solving and mathematics instruction, "whether problem

solving is intended as the end result of instruction or the means through which mathematical concepts, processes, and procedures are learned" (p.246). In other words, he viewed this relation from two perspectives, namely teaching for problem solving and teaching via problem solving and questioned which approach should be adopted by the teacher. Stein, Boaler, & Silver (2003) had first answered this question some years before that both approaches should be put together because both have their own merit. For example, teaching for problem solving would give the students, "an experience and/or knowledge of how and when to apply mathematical knowledge they have" (Nunokawa, 2005: p.329). The hidden objective of doing so is improving students' skill of familiarising (Hershkowitz, Schwarz, and Dreyfus, 2001). On the other hand, it seems to us that teaching via problem solving has the same connotation with what is called by Hiebert et al. (1996) as problematizing the subject where mathematics instruction is begun with problems, dilemmas and questions for students. By doing so, they believed that the students would be able to gain and develop their curiosity and sense-making skills. More specifically, it could be said that teaching via problem solving is another name for problem-based learning.

PROBLEM-BASED LEARNING (PBL)

Since being implemented fifty years ago, a debate regarding the effectiveness of problem-based learning as an instructional process emerged. Hung (2009) has reviewed both advocates and sceptics of PBL: on the one hand, this approach is more effective than traditional methods in improving students' problem solving and self-directed learning skills, but on the other hand it is costly and take more time from both students and teacher in order to gain similar learning outcomes. Additionally, as an advocate of PBL, Silver (2004) stated that it would be effective in assisting the students to become active learners because it utilises real world problems and encourages the students to become responsible for their learning. Although I do not really grasp what is meant by the sceptics of PBL of cost factors, I assume that the time consumption in PBL is caused by the wrong choice of problem that leads to ambiguity and difficulties for the students in solving the problems. This is supported by Willougby (1990, as cited in Williamson et.al, 2008) stating that poor problem-solving behaviours happen due to the presented problems, which are not authentic and do not accommodate the students to experience higher-order thinking processes.

Furthermore, Hung (2009) also suggested that the effectiveness of an instructional strategy cannot be determined by one criterion as it is a complex domain influenced by many variables. One of the variables mentioned as a determinant on the effectiveness of PBL is the design of the problem. In another article, Hung (2006) also contended that "problems, in general, are the heart of PBL" (p.56). This claim is obviously supported by Duch (2001) saying that the key success of PBL lies in the quality of the problem itself. Therefore, due to the fundamental role of a problem in determining the fruitfulness of PBL, some practitioners have proposed some principles to design PBL problems. For example, Dolmans et al. (1997) suggested seven principles to be considered, namely: students' prior knowledge, chance to do elaboration, relevant context, integration of knowledge, self-directed learning, and interest in the subject-matter and faculty objective. On the other hand, in order to guide the teachers or educators to create effective PBL problems for all level of learners, Hung (2006) propounded the 3C3R model. This model is built by two components, namely: core component and processing component, where the core component consists of, "…content, context and connection, [and are] used to support content/concept learning" (p.56), and

the processing components are elaborated into researching, reasoning and reflecting. These aspects play a role to enhance the cognitive process and problem solving skill of the students.

To summarise the comprehensive process of 3C3R in designing PBL problems, Hung (2009) composed nine steps for a design process, as follows, "(1) set goals and objectives; (2) conduct content/task analysis; (3) analyse context specification; (4) select/generate PBL problem; (5) conduct PBL problem affordance analysis; (6) conduct correspondence analysis; (7) conduct calibration process; (8) conduct reflection component; and (9) examine inter-supporting relationships of 3C3R components" (p.123). I believe that the powerful framework by Hung could become useful advice for teachers in developing problems for the promoter of applying PBL, because it combines the two main elements of the learning itself, that are the source (content in the relevant context and various connections) and the degree of skills required by the students in order to solve the problem (would the reasoning, researching and reflecting skills of the student be able to face the level of difficulty in the problem). The teachers might need a long time to be able to adopt this framework effectively, but I believe that if all mathematics teachers in the school

work together, monitored by the head, everything will be settled.

A tutorial process of PBL is proposed by Silver (2004) where the problem scenario would be presented to the students at the beginning of the learning process. Then, the formulating and analysing process of the problem would be started by identifying the relevant facts from the problem. Silver argued that the, "fact-identification step helps students represent the problem" (p.236). In order to do the identification process effectively, Polya (1957) suggested the teachers to ask the students about some beneficial questions, for instance: whether the students understand all the words used in presenting the problem; whether the students able to describe what are asked by the problem; and whether the students could restate the problem in their own word. Afterwards, as students' understanding toward the problems grows, they would be able to initiate the hypothesis of the possible solution. This hypothesis could be in the form of possible heuristic strategies to find the solution as previously mentioned. Silver (ibid) then added that getting the deficiency relative problem, learning issues that students research during their self-directed learning, is an important aspect while initiating the hypothesis. Following that, the students would apply the new knowledge and evaluate the hypothesis

based on the concepts that they have learned which this process was called by Polya (1957) as devising the plan. Lastly, students reflect on the abstraction of new knowledge that they have or looking back to the entire process (Polya, Ibid). Moreover, these processes also satisfy the terms of working mathematically proposed by Stacey (2005), which consists of six major activities, namely: investigating; conjecturing; using problem- solving strategies; applying and verifying; using mathematical language and working in context.

COMMUNICATION

Hiebert et al. (1996) assumed that, "understanding is the goal of mathematics instruction" (p.15). Furthermore, they looked at understanding from two points of view, namely: functional understanding and structural understanding. From a functional perspective, understanding will be gained while the students contribute and share their thoughts in the collective classroom activity; meanwhile, structural understanding is defined as what could be taken by the students from the classroom (Hiebert, 1996). Regarding these views, it seems to me that group work would support the students to get a comprehensive understanding because the students would share their ideas with each other while solving a mathematics problem in their groups. One

way to do a group work is known as cooperative learning which is defined by Felder & Brent (2007) as the process when the students work in groups to do some tasks or projects in which each member of the group is responsible for completing the tasks or projects. Additionally, Knuth and Peressini (2001) suggested the same notion about the importance of sharing opinions among the students in which, "the students will acquire a deeper understanding of mathematics when they use their own statement, as well as those of their peers and teacher" (p.325). Therefore, the role of a teacher is to accommodate a class setting where the students can discuss and share their ideas about mathematics and the teacher would listen to them (Sherin, 2000 as cited in Wichelt, 2009).

METACOGNITION

Seastone (1994) defined metacognition as, "one's ability to know what he knows and what he does not know" (Barbacena & Sy, 2005: p.17). Additionally, Aurah et al. (2011) suggested that metacognition is students' perceptions about three aspects: what she/he knows (metacognitive knowledge); what she/he can do (metacognitive skills); and what she/he knows about her/ his own cognitive abilities. It would be important for the teacher to know the metacognition ability of their students in order

to evaluate the teaching and learning process that has been done. By getting the information about what is and is not known by the students, the teacher would be able to decide any help that is needed and to what extent it is needed. Students themselves, by reflecting on the knowledge that they have grasped or not, will develop their confidence in learning and would be able to label themselves as a responsible learner and thinker (Barbacena & Sy, 2005). The strategies suggested by Barbacena & Sy to develop the metacognition skill of the students are reflective journal writing and cooperative learning. In cooperative learning, metacognition could happen while the students argue each other and reflect their thinking after sharing and listening to their friends. Writing the journal might work well to improve metacognition skill for some cases, but would the teacher be able to make sure whether what are written by the students are their real condition or not? There might be some children who cannot reveal their weakness obviously in the journal. Therefore, personal tutoring could be an additional option where the teacher and the student solve the problem together, and by doing so the teacher will know directly the current stage of students' understanding. However, this method might not be effective due to a large number of students.

THE NEW CURRICULUM OF INDONESIA

Indonesia has done the development of its curriculum from time to time. Katuuk (2014) argued that the change of the curriculum is a common phenomenon as it is one of the key elements in the educational process that could experience renewal as the society changes. So, the curriculum is changed to satisfy the needs of the society. Indonesia changes its curriculum due to two reasons: firstly, preparing the golden generations of Indonesia for a hundred years of independence; secondly, to tackle the problems which are associated with globalisation; transformation of the education sector; etc (Katuuk, 2014). In order to achieve these objectives, significant improvements have been made in the latest curriculum, such as: the concept of the curriculum, the assessment process, the learning process and the textbook (the Ministry of Education, Indonesia (Kemendikbud, 2014)). More importantly, the basic competence of learning has been focused on problem solving situated in real-world situations.

We believe that it is not easy for a teacher to implement a new curriculum following certain aspects which are unfamiliar to them. Some of the teachers might find it difficult to change their teaching habits from the previous curriculum to adapt to the new one as they used to implement it for several years.

Therefore, our intention is to introduce one of the approaches that can be used by the teacher to improve students' problem-solving skill which is one of the goals of the new curriculum. By applying this approach to teach mathematics, hopefully, the teachers could feel the sense which is meant by Indonesia's government in the new curriculum and the teachers could improve their students' problem-solving skill gradually.

THE FRAMEWORK OF PROBLEM SOLVING ORIENTED MATHEMATICS CURRICULUM (PSOMC)

The aim of the PSOMC is to create a teaching and learning process in gaining new mathematical knowledge, which positions problems at the centre. Therefore, the first thing that should be done by the teachers creates the meaningful problems that would allow the students to elaborate their thinking skills as effectively as possible in order to get a depth of understanding of the concepts. One approach to create a good problem is 3C3R which is proposed by Hung (2006). The acquisition of knowledge in this curriculum will be achieved by an applied PBL approach, where the students would engage with meaningful problems created in order to gain the new knowledge. The role of the teacher is to provide scaffolding that would guide the students to

achieve the learning goals. The students would work in groups where they can share and listen to others' ideas in order to get a depth of understanding. Following that, the teacher would encourage each group to present their work to all members of class where rich discussion would take place, resulting in complex understanding. Then, the students would evaluate their own learning through metacognitive processes that would allow them to grow to be confident and responsible learners. Furthermore, students' cognitive report would be informed to parents in order to maximise the assistance that can be delivered both in school and at home. On the other hand, students' attitudes through the learning process will be noted by the teachers as affective domain of assessment. In conclusion, the principles of PSOMC is presented in Diagram 1, as follow.
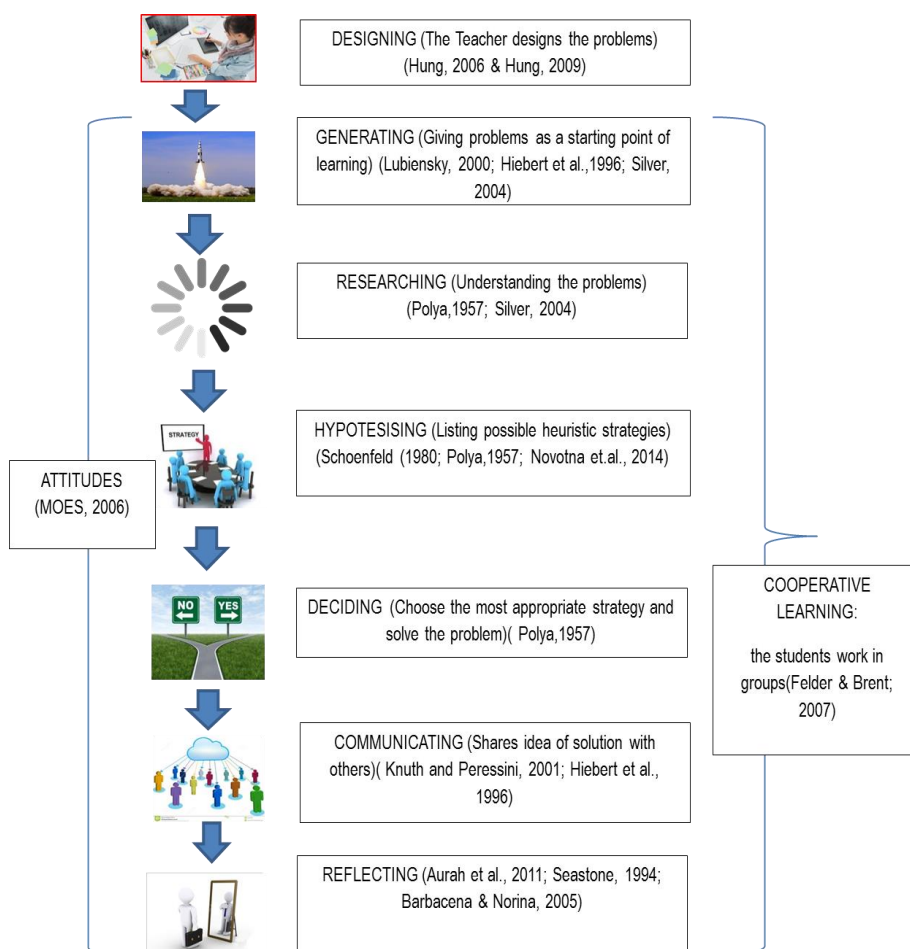

Figure 2. Principles of PSOMC

CONCLUSION

Mathematics as one of lesson that is the majority of students thinks as a difficult subject should be taught to stimulate students' problem-solving skills. Teachers who face many obstacales when teaching mathematics based on K13 (curriculum 2013) can consider to use specific cirriculum developed for teaching problem solving skills. Authors use literature review method for developing a framework of Problem Solving Oriented Mathematics Curriculum (PSOMC) that consist of seven priciples, those are designing, generating, researching, hipotesising, diciding, communicating, and reflecting.

Finally, through developing this curriculum, we hope teachers would be able to enhance the problem-solving skill of their students so that the number of the students that are frustrated with challenging problems would be decreased. We imagine a mathematics classroom which is full of confident learners to do problem-solving and they love to do so.

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